10 Orbit and Constellation Design—Selecting the Right Orbit

10.7 Design of Interplanetary Orbits

Faster Trajectories

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Using the recipe given in Table 10-29, one can compute the parameters of a Hohmann transfer between any pair of planets in a straightforward manner. Attractive as it is from an energy and computational point of view, a Hohmann transfer does have a number of drawbacks: (1) the launch epoch is very strict, with little or no room for deviations (at least ideally; pericenter and apocenter are by definition on opposite sides of the central body, with consequences for the departure and arrival epochs), (2) the time-of-flight may be too long for some applications, and (3) although it represents the most energy-efficient direct transfer, the total ΔV required for a direct trip to distant planets may simply be too large to be delivered by even the most powerful combination of launcher, upper stage and on-board engine (an indirect trajectory, involving planetary flybys, could provide a solution; more on this in Sec. 10.7.1).

In order to compensate for the first 2 drawbacks, one might consider to leave the departure planet (e.g., Earth) under conditions which are different from those of a Hohmann transfer; a similar flexibility can be introduced for the arrival geometry. This is the most general situation as sketched in Fig. 10-28. Clearly, our vehicle now follows an elliptical orbit with dimensions that exceed those of a Hohmann transfer orbit. As shown in Fig. 10-28, it can reach the orbit of the target planet on its way out, before reaching apocenter; in such a case we speak of a so-called Type-I transfer, which is clearly faster than a Hohmann transfer. In case the orbit of the target planet is reached after having passed the apocenter, we speak of a Type-II transfer, which has a longer transfer time compared to a Hohmann transfer. Both options (Type-I, Type-II) can have advantages and disadvantages (travel time, ΔV, departure, and arrival geometry, possibilities for continuation to another planet), so both should be considered.

In Table 10web-1, where use is made again of the patched conics approach, and where the departure conditions are given by the position of the departure planet, and a vectorial excess velocity $\boldsymbol{V}_{\infty,1}$. This excess velocity has a magnitude and a direction; the latter is specified by an angle α with respect to the tangential direction (note that it is different from the more common flight path angle ϕ). By virtue of the assumption of unperturbed Kepler orbits, all other parameters follow unambiguously. In general wording, this approach is called "forward propagation": define new initial conditions (e.g., at Earth) and compute the parameters when crossing the orbit of the target planet, at another epoch (the assumption of unperturbed Kepler orbits allows one to do so without doing a lengthy numerical integration). One warning is appropriate, though; one should check whether the conditions are favorable to reach the orbit of the target planet at all. If not, one should stop after Step 12 for an inbound mission and after Step 13 for an outbound mission; if needed, proceed with a new vector $\boldsymbol{V}_{\infty,1}$. Since we are dealing with circular orbits of the departure and target planets, the characteristics of the transfer orbit and related parameters only depend on the relative geometry of the two planets, the absolute positions do not matter. The epochs of departure and arrival can be computed with the equations given in Sec. 10.7.2, previously, with the travel time T_H replaced by T_{tr} and the transfer angle π (valid for a Hohmann transfer) replaced by $\Delta\nu = \nu_2 - \nu_1$ (the true anomaly of the satellite, at departure and arrival, respectively).

Comparing the values given in Tables 10-29 and 10web-1, it is clear that the transfer time can be reduced significantly; in this example, the reduction is by a factor of 3.5. However, this fast solution does not come for free: the amount of ΔV required increases by a factor of almost 4 (with major consequences for the amount of propellant, of course). The conditions of the example in Table 10web-1 are (deliberately) way off those for a Hohmann transfer; following the same recipe one can easily study the effect of minor variations with respect to the Hohmann conditions (in departure epoch, in ΔV_0, in launch direction).

Lambert Problem

In Sec. 10.7, two possibilities have been presented to travel from planet A to planet B: a Hohmann transfer and an arbitrary Kepler orbit (using the technique of forward propagation). The first option is easy but limited in results, whereas the second option leaves questions on the proper combination of the departure conditions: launch epoch, excess velocity and direction thereof. In reality, designers of interplanetary missions use a technique that reverses the formulation of the problem: assuming that the epochs of departure and arrival are known, the questions now become: what are the parameters of the Kepler orbit connecting the two? What is the corresponding ΔV? What are the departure and arrival geometries? What is the time-of-flight? This problem is generally referred to as the Lambert problem. In this web article, the essential formulas to solve this problem for high-thrust propulsion are presented. The method also serves as a good starter for the design of low-thrust trajectories. The theory for tackling the Lambert problem is taken from Gooding [1990].

Referring to Fig. 10web-1, the Lambert problem is easily formulated: knowing an initial position $\boldsymbol{r}_1$, a final

Table 10web-1. Summary of the Procedure to Compute the Characteristics of an Arbitrary Interplanetary Transfer. The meaning of the parameters is explained in the main body of the text. The example values hold for a trip from the Earth (at 1.0 AU) to Mars (at 1.52366 AU), with circular parking orbit altitudes of 185 and 500 km, respectively. The vehicle leaves (the sphere-of-influence of) the Earth with a $V_{\infty,1}$ of 10 km/s which is directed at an angle α of 15 deg with respect to the tangent of the circular orbit. The "*" sign indicates that care must be taken to choose the proper quadrant (use the atan2 function; see App. D.4); the "**" sign indicates that the proper sign of $V_{2,rad}$ must be chosen (depending on a Type-I or Type-II transfer).

Step	Parameter	Expression	Example
1	V_{dep} (heliocentric velocity of departure planet)	$V_{dep}=\sqrt{(\mu_S/r_{dep})}$	29.784 km/s
2	V_{tar} (heliocentric velocity of target planet)	$V_{tar}=\sqrt{(\mu_S/r_{tar})}$	24.129 km/s
3	V_{c0} (circular velocity around departure planet)	$V_{c0}=\sqrt{(\mu_{dep}/r_0)}$	7.793 km/s
4	V_{c3} (circular velocity around target planet)	$V_{c3}=\sqrt{(\mu_{tar}/r_3)}$	3.315 km/s
5	$V_{1,rad}$ (radial heliocentric velocity at departure position)	$V_{1,rad}=V_{\infty,1}\sin\alpha$	2.588 km/s
6	$V_{1,tan}$ (tangential heliocentric velocity at departure position)	$V_{1,tan}=V_{dep}+V_{\infty,1}\cos\alpha$	39.444 km/s
7	V_1 (heliocentric velocity at departure position)	$V_1=\sqrt{(V_{1,rad}^2+V_{1,tan}^2)}$	39.529 km/s
8	H (specific angular momentum)	$H=r_1V_{1,tan}$	59.01 10^8 km^2/s
9	ϕ_1 (flight path angle at departure position) (*)	$\phi_1=\mathrm{atan}(V_{1,rad}/V_{1,tan})$	3.754 deg
10	a_{tr} (semi-major axis of transfer orbit)	$a_{tr}=0.5\mu_S/(\mu_S/r_1-0.5V_1^2)$	6.27 10^8 km
11	e_{tr} (eccentricity of transfer orbit)	$e_{tr}=\sqrt{\left[1+2(H/\mu_S)^2(0.5V_1^2-\mu_S/r_1)\right]}$	0.763
12	r_p (pericenter distance)	$r_p=a_{tr}(1-e_{tr})$	1.49 10^8 km
13	r_a (apocenter distance)	$r_a=a_{tr}(1+e_{tr})$	11.05 10^8 km
14	V_2 (heliocentric velocity at target position)	$V_2\sqrt{(\mu_S(2/r_2-1/a_{tr}))}$	30.866 km/s
15	$V_{2,tan}$ (tangential heliocentric velocity at target position)	$V_{2,tan}=H/r_2$	25.887 km/s
16	$V_{2,rad}$ (radial heliocentric velocity at target position) (**)	$V_{2,rad}=\sqrt{(V_2^2-V_{2,tan}^2)}$	16.810 km/s
17	$V_{\infty,2}$ (excess velocity at target position)	$V_{\infty,2}=\sqrt{\left[V_{2,rad}^2+(V_{2,tan}-V_{tar})^2\right]}$	16.901 km/s
18	ϕ_2 (flight path angle at target position) (*)	$\phi_2=\mathrm{atan}(V_{2,rad}/V_{2,tan})$	32.997 deg
19	V_0 (velocity at pericenter of hyperbola around departure planet)	$V_0=\sqrt{(2\mu_{dep}/r_0+V_{\infty,1}^2)}$	14.882 km/s
20	V_3 (velocity at pericenter of hyperbola around target planet)	$V_3=\sqrt{(2\mu_{tar}/r_3+V_{\infty,2}^2)}$	17.540 km/s
21	ΔV_0 (maneuver at pericenter around departure planet)	$\Delta V_0=\lvert V_0-V_{c0}\rvert$	7.089 km/s
22	ΔV_3 (maneuver at pericenter around target planet)	$\Delta V_3=\lvert V_{c3}-V_3\rvert$	14.224 km/s
23	ΔV_{tot} (total velocity change)	$\Delta V_{tot}=\Delta V_0+\Delta V_3$	21.313 km/s
24	ν_1 (true anomaly at departure position) (*)	$\nu_1=\mathrm{atan}\left[V_{1,rad}H/(V_{1,tan}H-\mu_S)\right]$	8.680 deg
25	ν_2 (true anomaly at target position) (*)	$\nu_2=\mathrm{atan}\left[V_{2,rad}H/(V_{2,tan}H-\mu_S)\right]$	78.576 deg
26	E_1 (eccentric anomaly at departure position)	$E_1=2\mathrm{atan}\left[\sqrt{((1-e_{tr})/(1+e_{tr}))}\tan\nu_1/2\right]$	0.056 rad
27	E_2 (eccentric anomaly at target position)	$E_2=2\mathrm{atan}\left[\sqrt{((1-e_{tr})/(1+e_{tr}))}\tan\nu_2/2\right]$	0.583 rad
28	M_1 (mean anomaly at departure position)	$M_1=E_1-e\sin(E_1)$	0.013 rad
29	M_2 (mean anomaly at target position)	$M_2=E_2-e\sin(E_2)$	0.163 rad
30	T_{tr} (transfer time)	$T_{tr}=(M_2-M_1)/\sqrt{(\mu_S/a_{tr}^3)}$	6.47×10^6 s = 74.9 days = 0.205 yrs

Table 10web-2. Summary of the Procedure to Compute the Characteristics of a Kepler Orbit Connecting Two Arbitrary Points (i.e., the High-Thrust Lambert Problem). The meaning of the parameters is explained in the main body of the text. The example values refer to a hypothetical trip from the Earth (at 1.0 AU, parking orbit at 185 km) to Mars (at 1.52366 AU, parking orbit at 500 km), with a $\Delta\theta$ of 90 deg and a travel time Δt of 95 days. Step 9 (solution for "*x*") is typically done with a Newton-Raphson iteration (see [Gooding, 1990] for suggestions for an initial value of "*x*").

Step	Parameter	Expression	Example
1	V_{dep} (heliocentric velocity of departure planet)	$V_{dep} = \sqrt{(\mu_S / r_{dep})}$	29.784 km/s
2	V_{tar} (heliocentric velocity of target planet)	$V_{tar} = \sqrt{(\mu_S / r_{tar})}$	24.129 km/s
3	V_{c0} (circular velocity around departure planet)	$V_{c0} = \sqrt{(\mu_{dep} / r_0)}$	7.793 km/s
4	V_{c3} (circular velocity around target planet)	$V_{c3} = \sqrt{(\mu_{tar} / r_3)}$	3.315 km/s
5	*c* (chord)	$c = \sqrt{\left[r_1^2 + r_1^2 - 2 r_1 r_2 \cos(\Delta\theta)\right]}$	272.6 10^6 km
6	*s* (constant)	$s = (r_1 + r_2 + c) / 2$	325.1 10^6 km
7	*T* (normalized time-of-flight)	$T = \sqrt{(8\mu_S / s^3)}\, \Delta t$	1.443
8	*q* (constant)	$q = \sqrt{(r_1 r_2)} \cos(\Delta\theta / 2) / s$	0.402
9	*x* (new governing parameter), where	solution of $T - 2\,[x - qz(x) - d(x)/y(x))] / E_{lam}(x) = 0$	0.783
9a	E_{lam} (parameter)	$E_{lam} = x^2 - 1$	-0.386
9b	*y* (parameter)	$y = \sqrt{(\lvert E_{lam} \rvert)}$	0.621
9c	*z* (parameter)	$z = \sqrt{(1 - q^2 + q^2 x^2)}$	0.968
9d	*f* (parameter)	$f = y(z - qx)$	0.406
9e	*g* (parameter)	$g = xz - q\, E_{lam}$	0.914
9f	*d* (parameter)	$d = \text{atan}(f/g)$ (for $E_{lam}<0$); $d = \ln(f+g)$ (for $E_{lam}>0$)	0.418
10	a_{tr} (semi-major axis transfer orbit)	$a_{tr} = s / (2(1-x^2))$	4.208 10^8 km
11	γ (constant)	$\gamma = \sqrt{(\mu_S / 2)}$	4.644 10^9 km^2/s
12	ρ (constant)	$\rho = (r_1 - r_2) / c$	-0.287
13	σ (constant)	$\sigma = 2\sqrt{(r_1 r_2 / c^2)} \sin(\Delta\theta / 2)$	0.958
14	$V_{r,1}$ (radial velocity at departure)	$V_{r,1} = \gamma\,[(qz-x) - \rho(qz+x)] / r_1$	-1.789 km/s
15	$V_{r,2}$ (radial velocity at target)	$V_{r,2} = -\gamma\,[(qz-x) + \rho(qz+x)] / r_2$	14.902 km/s
16	$V_{tan,1}$ (tangential velocity at departure)	$V_{tan,1} = \gamma\,\sigma\,(z+qx) / r_1$	38.153 km/s
17	$V_{tan,2}$ (tangential velocity at target)	$V_{tan,2} = \gamma\,\sigma\,(z+qx) / r_2$	25.041 km/s
18	$V_{\infty,1}$ (excess velocity at departure position)	$V_{\infty,1} = \sqrt{\left[V_{1,rad}^2 + (V_{1,tan} - V_{dep})^2\right]}$	8.558 km/s
19	$V_{\infty,2}$ (excess velocity at target position)	$V_{\infty,2} = \sqrt{\left[V_{2,rad}^2 + (V_{2,tan} - V_{tar})^2\right]}$	14.930 km/s
20	V_0 (velocity at pericenter of hyperbola around departure planet)	$V_0 = \sqrt{(2\mu_{dep} / r_0 + V_{\infty,1}^2)}$	13.954 km/s
21	V_3 (velocity at pericenter of hyperbola around target planet)	$V_3 = \sqrt{(2\mu_{tar} / r_3 + V_{\infty,2}^2)}$	15.649 km/s
22	ΔV_0 (maneuver at pericenter around departure planet)	$\Delta V_0 = \lvert V_0 - V_{c0} \rvert$	6.161 km/s
23	ΔV_3 (maneuver at pericenter around target planet)	$\Delta V_3 = \lvert V_{c3} - V_3 \rvert$	12.334 km/s
24	ΔV_{tot} (total velocity change)	$\Delta V_{tot} = \Delta V_0 + \Delta V_3$	18.494 km/s

position $\boldsymbol{r_2}$ (note: both expressed as vectors in space), and the time-of-flight Δt between the two positions, what are the specifics of the Kepler orbit connecting the two positions? For the sake of brevity, we will only consider the situation where our vehicle travels from P_1 to P_2 in a counterclockwise direction, and does so directly, as illustrated in Fig. 10web-1 (i.e., without completing more than a full orbit). For a discussion on the more general case, refer to Gooding [1990].

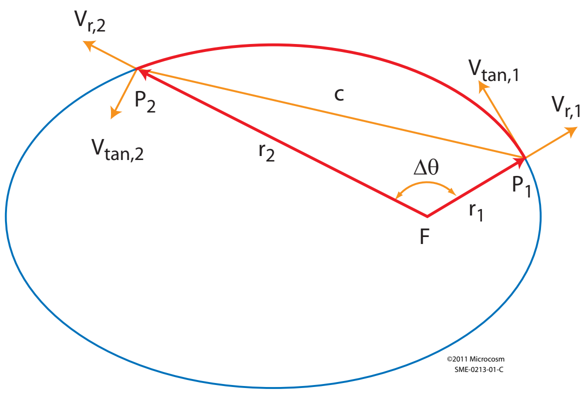

Fig. 10web-1. Illustration of Lambert Problem.

The expressions used to derive the solution, including the heliocentric velocities at P_1 and P_2, are summarized in Table 10web-2. Steps 1–4 relate to the velocities of the planets and to the circular orbits around them, and are common with those of the Hohmann and forward propagation techniques (see Tables 10-29 and 10web-1); the essential formulations to solve the Lambert problem are given in steps 5–17. The table is completed with a numerical example, again for an arbitrary transfer between the Earth and Mars. In this example, the departure and arrival vectors are assumed to be at right angles ($\Delta\theta = 90$ deg), and the required trip time is hypothesized to be 95 days. More realistic combinations of $\Delta\theta$ and trip time would follow directly from the user's model for the planetary ephemerides: assuming departure and arrival epochs (e.g., depart from the Earth on May 1, 2015, and arrive at Mars on October 17, 2015) gives the positions of the planets, including the angle $\Delta\theta$ in between, directly. According to this example, wanting to cover 90 deg in a mere 95 days is a bad idea: it would require a total ΔV of 18.494 km/s, which although better than the (also arbitrary) example in Table 10web-1, is still much worse than the Hohmann transfer illustrated in Table 10-29.

Apparently, we introduced a rather bad example in Table 10web-2; the transfer would still cost us a total ΔV of 18.5 km/s. However, the Lambert formulation has one huge advantage: it provides the opportunity to assess a wide range of trajectory options rapidly. By varying the departure and arrival epochs (which then directly give the positions of the departure and target planets, hence $\Delta\theta$, and the trip time), one can easily identify a range of corresponding ΔV values and select the settings (i.e., epochs) of the most attractive solution (in the sense of minimum ΔV, or geometrical conditions). As general advice, one could start with the computation of the conditions for a Hohmann transfer first (including the determination of the launch and arrival epochs, using Table 10-29 and the equations in the section on "Timing"), and then use the Lambert technique to search for the real optimum using refined, 3-dimensional planetary ephemerides, or assess the sensitivities of a first-order solution by using the recipe in Table 10web-2. Since the outcome of a Lambert solution is driven by the dates of departure and arrival (i.e., translated into the positions of the departure and target planets plus the time of flight), it is appropriate to give an advise for the interval in which the optimal combination of epochs t_1 and t_2 can be found. In case of a direct flight from Earth to Mars, searching in a window of about 50 days around the epochs that come out of the Hohmann solution should be fine. For trips to inner planets one can do with a smaller search interval, whereas for trips to more distant planets, a somewhat larger window is to be used. In case that our proposed mission scenario takes the vehicle along a succession of planets (i.e., planetary flybys; see main text), the entire mission consists of a series of so-called "legs" and the search intervals for the flyby epochs should become larger.

Pork Chop Plots

To identify the best possible trajectory, one can consider to launch and arrive on a range of dates, and see what to pay (or possibly gain) in terms of ΔV and time-of-flight for all possible pairs of dates. Typically, this is done using a straightforward application of the Lambert formulation as presented in Table 10web-2. Often, the results of the various trajectory proposals are used in an optimization. This topic will not be discussed in this chapter; refer to standard works on the matter such as Kirk [1998], Lawler and Wood [1966], Munkres [1957], Myatt et al. [2003], Ross and D'Souza [2005], Winston [2004], Conway [2010] and Kemble [2006], or publications on popular enumerative optimization techniques (e.g., Goldberg [1989], Michalewicz [1996], Kennedy and Eberhart [1995], Wilke [2005] Price et al. [2005], Feoktistov [2006], Kirkpatrick et al [1983]). Here, we will give some results only. As an example, Fig. 10web-2 illustrates the ΔV and the time-of-flight for one-way trips between the Earth and Mars (starting from a parking orbit at 185 km altitude above the Earth and ending up in a circular orbit at 500 km above Mars' surface). In the calculations, the date of departure is successively run through the interval January 1, 2010 to January 1, 2020, and for each departure date the most attractive flight (in terms of minimum ΔV) is shown in the plot on the left. The corresponding time-of-flight is instead shown in the plot on the right. The calculations have been done for real ephemerides of the planets, taken from JPL's DE200 Standish [1982] (a more recent version is DE405 Standish [1998]).

First of all, Fig. 10web-2 clearly shows the effect of the planetary geometry on such a transfer: in some situations we can fly the mission with relatively little ΔV,

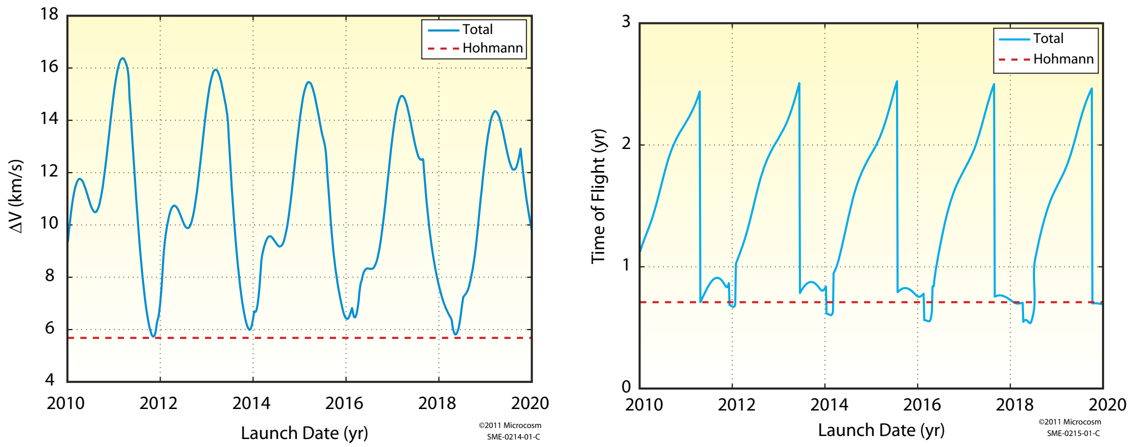

Fig. 10web-2. ΔV (Left) and Time-of-Flight (Right) Required for a One-Way Transfer Between an Orbit at 185 km Above the Earth and an Orbit at 500 km above Mars' Surface, as a Function of the Launch Date.

whereas in other situations, the planetary configuration is very unattractive. Clearly, the optima repeat after slightly more than 2 years, nicely matching the synodic period whose precise value is included in Tables 9-5 and 9-6. An idealized Hohmann transfer between Earth (at 1 AU) and Mars (at 1.52366 AU) would take a ΔV of 5.68 km/s, and a time-of-flight of 0.709 yr (see Table 10-29). Fig. 10web-2 also shows the effect of the real ephemerides of the two planets: for example, the values for the optima (i.e., the minimum values) show some variation, meaning that in some years it is more attractive to do this transfer than it is in other years.

A typical approach to get more insight into such problems is to generate (or use, for that matter) pork chop plots. In such plots, the ΔV is given as a function of departure date and flight time. As an example, Fig. 10web-3 shows a pork chop plot for the same transfer as in Fig. 10web-2. The situation where the total ΔV takes a minimum value corresponds with a Hohmann transfer.

One can easily compute such pork chop plots, using the theory presented in Table 10web-2 (it is a straightforward end-to-end computation, driven by a range of possible dates for departure from Earth and for arrival at Mars). An alternative is to use the information that is available in publications such as Sergeyevsky and Cunniff [1987]. Note that in Figs. 10web-2 and 10web-3, we have shown the total amount of ΔV, which is a quantity that needs to be produced physically by the high-thrust rocket engine (remember: $\Delta V_{tot} = |\Delta V_0| + |\Delta V_3|$, see Tables 10-29 and 10web-2). In literature, you may find similar plots which show different parameters, such as the value of C_3 at departure ($C_3 = V_\infty^2$). However, the actual (propellant-demanding) acceleration and deceleration are represented by the maneuvers ΔV_0 and ΔV_3, and are given in the two parking orbits. As mentioned earlier, they can differ significantly from the excess velocities V_∞.

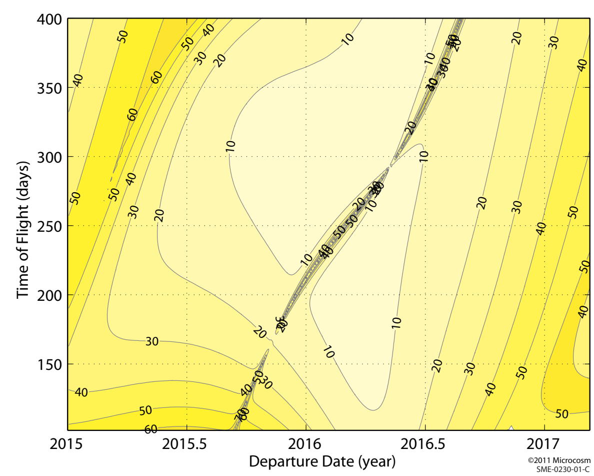

Fig. 10web-3. Pork Chop Plot of the Total ΔV for a Transfer from the Earth (parking orbit at 185 km) to Mars (target orbit at 500 km altitude).