Appendix E

Time and Date Systems

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Time is something we intuitively understand extremely well and, consequently, understand poorly in detail. For example, any phenomenon which repeats on a regular basis, such as a pendulum or the motion of electrons in an atom, can be used as the basis for timekeeping. However, "regularly repeating" implies that we have some standard to compare with. The only way to determine a clock's accuracy is to compare it with another clock, which of course leads to the question of which clock we should assume to be correct. Historically, the fundamental "clocks" chosen for maintaining absolute time have been the rotation of the Earth on its axis and the revolution of the Earth about the Sun. However, modern time measurements have shown these systems to be both nonuniform and extremely difficult to model with precision. For example, Fig. B-3 in App. B shows the variations in the length of the day over the past several centuries. On average, the Earth is slowing down by 1.5 ms/century but local variations are very substantial.*

Unfortunately, clock errors are cumulative, and therefore, high accuracy can prove important. For example, assume a spacecraft in low Earth orbit has a clock that is slowing down by 1 ms/day, which is small enough to be difficult to measure by many processes. This would result in a cumulative error of 0.365 sec over a year which corresponds to an error in position of the satellite of 3 km. Thus, if we wish to know the positions of satellites to tens of meters, we will need reasonably good clocks to do so.

Two basic types of time measurement are used in spacecraft systems: (1) *time intervals* between 2 events such as the spacecraft's spin period, orbital period, or the length of time a sensor sees the Earth; and (2) *absolute times* or *calendar times* of specific events, such as the time associated with some particular spacecraft observation. Of course, calendar time is simply a time interval for which the beginning event is an agreed standard.

E.1 Calendar Time and Long Duration Intervals

Calendar time in the usual form of date and time is used only for input and output because generally, arithmetic is cumbersome in months, days, hours, minutes, and seconds (for computations, absolute time is used instead, as explained next). Nonetheless, this is used for most human interaction with space systems because it's the system with which we are most familiar. Even with date and time systems, problems can arise, because time zones are different throughout the world and spacecraft operations typically involve a worldwide network. The uniformly adopted solution to this problem is to use the local standard time corresponding to 0 deg longitude (i.e., the Greenwich meridian) as the assigned time for events anywhere in the world or in space. This is referred to as *Universal Time* (UT), *Greenwich Mean Time* (GMT), or *Zulu* (Z), all of which are equivalent for most practical spacecraft operations. For precise computations, UT has replaced GMT, since the term GMT can be ambiguous [Seidelmann, 2006]. The name Greenwich Mean Time was chosen because 0 deg longitude is defined by the site of the former Royal Greenwich Observatory southeast of central London.

Civil time, T_{civil}, as measured by a standard wall clock or time signals, differs from Universal Time by an integral number of hours, corresponding approximately to the longitude of the observer. The approximate relation is:

$$T_{civil} \approx UT \pm L / 15 \qquad \text{(E-1)}$$

where T_{civil} and UT are in hours, and L is the longitude in degrees with the plus sign corresponding to East longitude and the minus sign corresponding to West longitude.

The conversion between civil time and Universal Time for most North American and European time zones is given in Table Eweb-1. Substantial variations in time zones are created for political convenience. In addition, most of the United States and Canada observe Daylight Savings Time from the second Sunday in March until the first Sunday in November†. Most European countries observe Daylight Savings Time (called "Summer Time") from the last Sunday in March to the last Sunday in October. Many countries in the southern hemisphere also maintain Daylight Savings Time, typically from October to March. Countries near the equator typically do not deviate from standard time.

Calendar time is remarkably inconvenient for computation, particularly over long time intervals of months or years. We need an absolute time that is a continuous count of time units from some arbitrary reference. The

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* The overall slowing of the Earth's rotation and lengthening of the day are caused primarily by tidal friction with the Moon. Local variations are caused principally by the growth and decline in the polar ice caps which shifts large quantities of water from the pole (smaller moment of inertia and higher spin rate) to the equator (larger moment of inertia and lower spin rate).

† From 1987 to 2006, Daylight Savings Time in the United States began on the first Sunday of April and ended on the last Sunday of October. By the Energy Policy Act of 2005, Daylight Savings Time was extended starting 2007. The Canadian system follows that of the United States.

Table Eweb-1. Time Zones in North America, Europe, and Japan. In most of the United States, Daylight Savings Time is used from the first Sunday in April until the last Sunday in October. In Europe, the equivalent "Summer Time" is used from the last Sunday in March to the first Sunday in October.

Time Zone	Standard Meridian (Deg, East Long.)	UT Minus Standard Time (Hours)	UT Minus Daylight Time (Hours)
Atlantic	300	4	3
Eastern	285	5	4
Central	270	6	5
Mountain	255	7	6
Pacific	240	8	7
Alaska	225	9	8
Hawaii	210	10	NA
Japan	135	–9	NA
Central Europe	15	–1	–2
United Kingdom	0	0	–1

time interval between any two events is then found by simply subtracting the absolute time of the second event from that of the first. The universally adopted solution for astronomical problems is the *Julian Date*, JD, a continuous count of the number of days since Greenwich noon (12:00 UT) on January 1, 4713 BC,* or, as astronomers now say, –4712. Because Julian Days start at noon UT, they will be a half day off with respect to civil dates. While this is inconvenient for transforming from civil dates to Julian Dates, it was useful for astronomers because this way the date didn't change in the middle of the night (for European observers).

As described below, there are 4 general approaches for converting between calendar dates and Julian Dates.

Table Look-Up

Tabulations of the current Julian Date are in most astronomical ephemerides and almanacs. Table E-1 lists the Julian Dates at the beginning of each year from 2000 through 2041. To find the Julian Date for any given calendar date, simply add the *day number* within the year (and fractional day number, if appropriate) to the Julian Date for Jan 0.0 of that year from Table E-2. Day numbers for each day of the year are on many calendars or can be found by adding the date to the day number for day 0 of the month from Table E-2. Thus 18:00 UT on April 15, 2014 = day number 15.75 + 90 = 105.75 in 2014 = JD 105.75 + 2,456,657.5 = JD 2,456,763.25.

* This strange starting point was suggested by an Italian scholar of Greek and Hebrew, Joseph Scaliger, in 1582 as the beginning of the current *Julian period* of 7,980 years. This period is the product of three numbers: the *solar cycle*, or the interval at which all dates recur on the same days of the week (28 years); the *lunar cycle*, containing an integral number of lunar months (19 years); and the *indiction* or the tax period introduced by the Emperor Constantine in 313 AD (15 years). The last time that these started together was 4713 BC and the next time will be 3267 AD. Scaliger was interested in reducing the astronomical dating problems associated with calendar reforms of his time and his proposal had the convenient selling point that it pre-dated the ecclesiastically approved date of creation, October 4, 4004 BC.

To convert from Julian Days to dates, determine the year in which the Julian Date falls from Table E-1. Subtract the Julian Date from the JD for January 0.0 of that year to determine the day number within the year. This can be converted to a date (and time, if appropriate) by using day numbers on a calendar or subtracting from the day number for the beginning of the appropriate month from Table E-2. Thus, from Table E-1, JD 2,456,073.25 is in the year 2012. The day number is 2,456,073.25 – 2,455,926.5 = 146.75. From Table E-2, this is 18:00 UT, May 25, 2012.

Software Routines Using Integer Arithmetic

A particularly clever procedure for finding the Julian Date, JD, associated with any current year, Y, month, M, and day of the month, D, is given by Fliegel and Van Flandern [1968] as a computer statement using integer arithmetic. Note that all of the variables must be defined as integers (i.e., any remainder after a division must be truncated) and that both the order of the computations and the parentheses are critical. This procedure works in FORTRAN, C, C++, and Ada for any date on the Gregorian calendar that yields JD > 0. (Add 10 days to the JD for dates on the Julian calendar prior to 1582.) The formula is:

$$\begin{aligned} JD_0 = {} & D - 32{,}075 + 1461 \times (Y + 4800 \\ & + (M - 14) / 12) / 4 + 367 \times (M - 2 \\ & - (M - 14) / 12 \times 12) / 12 - 3 \times ((Y + 4900 \\ & + (M - 14) / 12) / 100) / 4 \end{aligned} \qquad \text{(Eweb-1a)}$$

Here JD_0 is the Julian Day beginning at noon UT on the given date and must be an integer. For a fractional day, F, in UT (i.e., day number "D.F"), the floating point Julian Day is given by:

$$JD = JD_0 + F - 0.5 \qquad \text{(Eweb-1b)}$$

For example, the Julian Day beginning at 12:00 UT on December 25, 2015 (Y = 2015, M = 12, D = 25) is JD 2,457,382 and 6:00 UT on that date (F = 0.25) is JD 2,457,381.75.

Table E-1. Julian Date at the Beginning of Each Year from 2000 to 2041. See text for explanation of use. The day number for the beginning of the year is called "Jan. 0.0" (actually Dec. 31st of the preceding year) so that day numbers can be found by simply using dates. Thus, Jan. 1 is day number 1 and has a JD 1 greater than that for Jan. 0. * = leap year.

Year	*JD* 2,400,000+ for Jan 0.0 *UT*	Year	*JD* 2,400,000+ for Jan 0.0 *UT*	Year	*JD* 2,400,000+ for Jan 0.0 *UT*
2000*	51,543.5	**2014**	56,657.5	**2028***	61,770.5
2001	51,909.5	**2015**	57,022.5	**2029**	62,136.5
2002	52,274.5	**2016***	57,387.5	**2030**	62,501.5
2003	52,639.5	**2017**	57,753.5	**2031**	62,866.5
2004*	53,004.5	**2018**	58,118.5	**2032**	63,231.5
2005	53,370.5	**2019**	58,483.5	**2033**	63,597.5
2006	53,735.5	**2020***	58,848.5	**2034**	63,962.5
2007	54,100.5	**2021**	59,214.5	**2035**	64,327.5
2008*	54,465.5	**2022**	59,579.5	**2036**	64,692.5
2009	54,831.5	**2023**	59,944.5	**2037**	65,058.5
2010	55,196.5	**2024***	60,309.5	**2038**	65,423.5
2011	55,561.5	**2025**	60,675.5	**2039**	65,788.5
2012*	55,926.5	**2026**	61,040.5	**2040**	66,153.5
2013	56,292.5	**2027**	61,405.5	**2041**	66,519.5

Table E-2. Day Numbers for Day 0.0 of Each Month. Leap years (in which February has 29 days) are those evenly divisible by 4. However, years evenly divisible by 100 are not leap years, except that those evenly divisible by 400 are. Leap years are indicated by * in Table E-2.

Month	Non-Leap Years	Leap Years
January	0	0
February	31	31
March	59	60
April	90	91
May	120	121
June	151	152
July	181	182
August	212	213
September	243	244
October	273	274
November	304	305
December	334	335

The inverse routine for computing the date from the Julian Day is given by:

$$L = JD_0 + 68{,}569 \qquad \text{(Eweb-2a)}$$

$$N = (4 \times L) / 146{,}097 \qquad \text{(Eweb-2b)}$$

$$L = L - (146097 \times N + 3) / 4 \qquad \text{(Eweb-2c)}$$

$$I = (4000 \times (L + 1)) / 1{,}461{,}001 \qquad \text{(Eweb-2d)}$$

$$L = L - (1461 \times I) / 4 + 31 \qquad \text{(Eweb-2e)}$$

$$J = (80 \times L) / 2{,}447 \qquad \text{(Eweb-2f)}$$

$$D = L - (2447 \times J) / 80 \qquad \text{(Eweb-2g)}$$

$$L = J / 11 \qquad \text{(Eweb-2h)}$$

$$M = J + 2 - 12 \times L \qquad \text{(Eweb-2i)}$$

$$Y = 100 \times (N - 49) + I + L \qquad \text{(Eweb-2j)}$$

where integer arithmetic is used throughout. Y, M, and D are the year, month, and day, and I, J, L, and N are intermediate variables. Finally, again using integer arithmetic, the day of the week, W, corresponding to the Julian Date beginning at 12:00 on that day is given by:

$$W = JD_0 - 7 \times ((JD + 1) / 7) + 2 \qquad \text{(Eweb-3)}$$

where W = 1 corresponds to Sunday. Thus, December 25, 2015 falls on Friday.

Software Routines Without Integer Arithmetic

While most computer languages provide integer arithmetic, common software tools such as Excel or MatLab typically do not. (See below for use of Excel and MatLab DATE functions.) Similar capabilities are available using integer (INT) or truncation (TRUNC in Excel, FIX in MatLab) functions. INT and TRUNC are identical for positive numbers, but differ for negative numbers: INT (–3.1) = –4, whereas TRUNC (–3.1) = –3. It is the TRUNC or FIX function which is equivalent to integer arithmetic. Thus, using the same variables as above, we can rewrite Eqs. (Eweb-1a and Eweb-1b) for computation of JD from the date as:

$$C = \text{TRUNC}((M - 14) / 12) \qquad \text{(Eweb-4a)}$$

$$JD_0 = D - 32{,}075 + \text{TRUNC}(1{,}461 \times (Y + 4{,}800 + C) / 4) + \text{TRUNC}(367 \times (M - 2 - C \times 12) / 12) - \text{TRUNC}(3 \times (\text{TRUNC}(Y + 4{,}900 + C) / 100) / 4) \quad \text{(Eweb-4b)}$$

$$JD = JD_0 + F - 0.5 \quad \text{(Eweb-4c)}$$

where again JD_0, Y, M, D, and C are integers and F and JD are real numbers. Applying the same rules to Eqs. (Eweb-2a–Eweb-2j) gives the inverse formula for the date in terms of JD as:

$$L = JD + 68{,}569 \quad \text{(Eweb-5a)}$$

$$N = \text{TRUNC}((4 \times L) / 146{,}097) \quad \text{(Eweb-5b)}$$

$$L = L - \text{TRUNC}((146097 \times N + 3) / 4) \quad \text{(Eweb-5c)}$$

$$I = \text{TRUNC}((4000 \times (L + 1)) / 1{,}461{,}001) \quad \text{(Eweb-5d)}$$

$$L = L - \text{TRUNC}((1{,}461 \times I) / 4) + 31 \quad \text{(Eweb-5e)}$$

$$J = \text{TRUNC}((80 \times L) / 2{,}447) \quad \text{(Eweb-5f)}$$

$$D = L - \text{TRUNC}((2{,}447 \times J) / 80) \quad \text{(Eweb-5g)}$$

$$L = \text{TRUNC}(J / 11) \quad \text{(Eweb-5h)}$$

$$M = J + 2 - 12 \times L \quad \text{(Eweb-5i)}$$

$$Y = 100 \times (N - 49) + I + L \quad \text{(Eweb-5j)}$$

where the variables are the same as Eq. (Eweb-2a–Eweb-2j), except that D is now a real number corresponding to the date and fraction of a day. Finally, Eq. (Eweb-3) for the day of the week becomes:

$$W = JD - 7 \times \text{TRUNC}((JD + 1.5) / 7) + 2.5 = JD - 7 \times \text{INT}((JD + 1.5) / 7) + 2.5 \quad \text{(Eweb-6)}$$

where $1 \le W < 2$ corresponds to Sunday. The examples given above can also serve as test cases for Eqs. (Eweb-4a–Eweb-4c), (Eweb-5a–Eweb-5j), and (Eweb-6).

Modified Julian Date

The Julian Date presents minor problems for space applications. Because it was introduced principally for astronomical use, Julian Dates begin at 12:00 UT rather than 0 hours UT, as the civil calendar does (thus the 0.5 day differences in Table E-1). In addition, the 7 digits required for the Julian Date did not permit the use of single precision arithmetic in older computer programs. This is no longer a problem with modern computer storage and number formats. Nonetheless, various forms of truncated Julian dates have gained at least some use.

The most common of the truncated Julian Dates for astronomy and astronautics use is the *Modified Julian Date*, MJD, given by:

$$MJD = JD - 2{,}400{,}000.5 \quad \text{(Eweb-7)}$$

MJD begins at midnight, to correspond with the civil calendar. Thus, in using Table E-1, MJD is given by adding the day of the year (plus fractions of a day, if appropriate) to the number in the table, with the ".5" at the end of the table-listing dropped. For example, MJD for 18:00 UT on Jan. 3, 2016 = MJD 57,390.75. The definition of MJD given here is that adopted by the International Astronomical Union in 1997. Note, however, that other definitions of MJD have been used. Thus, the most unambiguous approach remains the use of the full Julian Date.

Spreadsheets such as Excel or Matlab

Spreadsheets, such as Excel or Matlab, typically store dates internally as some form of day count from a certain predetermined date. We call K the Julian Date expression of such predetermined "Day 0" date. Thus, we can either subtract two dates directly to determine a time interval or convert them to Julian Dates and then make the subtraction. It's possible to convert them to JD by simply finding the additive constant, K, given by:

$$K = JD - I \quad \text{(Eweb-8)}$$

where I is the internal number representing a specific known date, JD. Once K is determined, then the JD for any date is:

$$JD = K + I \quad \text{(Eweb-9)}$$

Many versions of Excel use Jan. 1, 1900, as "day 0," such that K_{Excel} = 2,415,020.5. However, this should be checked for individual programs because other starting points are sometimes used and the starting point is a variable parameter in some versions of Excel. While this can be a very convenient function, Excel date routines run only from 1900 to 9999.

Matlab typically uses Jan. 1, 0000, 0:0:0 as "day 0." Thus, in the formula above, K_{Matlab} = 1,721,058.5.

Any of the day counting approaches will work successfully over its allowed range. However, systems intended for general mathematics or business use may not account correctly for leap years and calendar changes when historical times or times far in the future are being evaluated. Thus, the use of the full Julian Date remains the most unambiguous solution, particularly if a program or result is to be used by more than one person or activity. For a more extended discussion of time systems, see Seidelmann [2006].

E.2 Modern Time Measurement —Short Duration Intervals

As one would expect, modern technology has lead to ever increasing precision in the measurement of time. However, in addition, new processes for measuring time have been introduced, as well as new and fundamentally different definitions of the meaning of time in both science and engineering. In the 1950s, *Ephemeris Time*, *ET*, was introduced, based on the dynamic equations of motion of the Earth. For many years, this was used as the basis of astronomical and astrophysical ephemerides, i.e., the most precise orbit calculations. In 1967, the second was redefined as having an atomic standard but ephemeris time remained in use for the motion of planets and satellites. In 1984, ephemeris time was unceremoniously abandoned in favor of *Terrestrial Dynamic Time, TDT*, in which the unit

Table E-3. Common Time Systems.

Kind of Time	Defined By	Fundamental Unit	Regularity	Application
Sidereal (ST)	Earth's rotation relative to stars	Sidereal day, 1 rotation of Earth with respect to stars	Irregular	Astronomical observations; determining *UT* and rotational orientation of Earth
Solar				
Apparent	Earth's rotation relative to true Sun	Successive transits of Sun	Irregular and annual variations	Sundials*
Mean	Earth's rotation relative to fictitious mean Sun	Mean solar day	Uniform and annual variations	Confuse students and engineers; use uniform time
Universal				
UT0	Observed *UT*	Mean solar day	Irregular	Study of Earth's wandering pole
UT1	Corrected *UT0*	Mean solar day	Irregular	Shows seasonal variation of Earth's rotation
UT2	Corrected *UT1*	Mean solar day	Irregular	Basic rotation of Earth
UTC=GMT=Z	Atomic sec and leap sec to approximate *UT1*	Mean solar day	Uniform except for leap sec	Civil timekeeping; terrestrial navigation and surveying; broadcast time signals
Ephemeris (ET)	Fraction of tropical year 1900	Ephemeris sec	Uniform	Ephemerides prior to 1994; no longer in use
Atomic (TAI)	Frequency of Ce-133 radiation	Atomic sec = Ephemeris sec	Uniform	Basis of *ET* and *UTC*
GPS	Atomic sec without leap sec	Atomic sec	Uniform	Time component of GPS signals
Relativistic				
Terrestrial (TT)	Atomic sec at mean sea level on Earth	Atomic sec at Earth's surface	Uniform	Ephemerides
Barycentric Dynamic (TDB)	Orbital equations of motion with respect to barycenter of the Solar System	Atomic sec adjusted for relativistic effects	Uniform	Transforms Earth-based time to time kept by the motions of the planets

* Devices showing the time of day by the shadow of a pointer (called style or gnomon) cast by the Sun on a plate or surface marked with lines that indicate the hours of the day.

of measure was the atomic second and the scale was chosen to agree with ephemeris time in 1984. In 1991, the general theory of relativity was explicitly adopted as the theoretical background for defining both space and time reference frames, *TDT* was renamed *Terrestrial Time, TT*, and the definition of the second was "adjusted" to correspond to atomic measurements at a specific location (i.e., at mean sea level on the surface of the Earth).*

Currently, there are 4 main types of time systems in use:

- *Atomic Time, TAI*, for which the unit of duration corresponds to a defined number of wavelengths for a specific atomic transition of a specific isotope
- *Universal Time, UT*, for which the unit of duration is the rotation of the Earth with respect to the Sun, defined to be as uniform as possible despite variations in the physical rotation rate of the Earth
- *Sidereal Time, ST*, for which the unit of duration is the rotation of the Earth with respect to the vernal equinox which, in turn, is nearly fixed with respect to the mean positions of the stars
- *Dynamical Time, DT*, for which the unit of duration is based on the orbital motion of the Earth, Moon, and planets. Terrestrial Time, *TT*, belongs to this family of timescales

* An excellent discussion of the history of time systems is provided by Seidelmann [2006].

In addition, rapidly rotating pulsars may provide an even more accurate standard for future time systems. While the differences between the various time systems are subtle, they can have important implications for spacecraft systems and applications. Commonly used modern time systems are defined in Table E-3.

Fortunately, the basis for all of the modern time systems is the Système International, SI, *second*. This is defined as the duration of 9,192,631,770 periods of the radiation corresponding to the transition between 2 hyperfine levels of the ground state of the Cesium-133 atom measured at mean sea level on the Earth. This definition of the second corresponds more-or-less to 1 / 86,400 (= 1 / [60 × 60 × 24]) of the rotation period of the Earth, relative to the mean Sun. It is, of course, the "more-or-less" part which ultimately causes most of the problems in time measurement systems. Some of the time systems are described in more detail below.

Atomic Time (TAI)

International Atomic Time, TAI (*Temps Atomique International*), is a practical implementation of a time standard based on the SI second. An excellent approximation to *TAI* can be maintained by laboratory Cesium clocks. A large number of such clocks are compared from time to time and a weighted average is prepared which provides a fine adjustment for each of the individual clocks. The use of these Cesium-based clocks provides a readily available basis for timekeeping for all types of physical, astronomical, and space-related observation.

Universal Time (UT)

Universal Time, UT, follows the irregular rotation of the Earth, and is often referred to as a type of solar time because the objective in universal time is to remain synchronized with the orientation of the Earth relative to the Sun. The most important subcategory of universal time is *Coordinated Universal Time (UTC),* which is the basis for civil timekeeping and broadcast time signals worldwide. *UTC* uses the SI second as the basic unit of time and then adds (or, in principle, subtracts) a leap second at the end of the last day of June and the last day of December as needed to maintain close agreement with the rotational orientation of the Earth. Thus, *UTC* lags behind *TAI* by an integral number of seconds. For example, for January 1, 1996, *TAI* minus *UTC* equaled 30 sec exactly. There is a long-term continuous drift between *UTC* and *TAI* that cannot be predicted in advance, because *UTC* takes into account irregularities in the motion of the Earth.

In applications where precision is not critical, *UTC* frequently goes by the name Greenwich Mean Time (*GMT*), Zulu (*Z*), or simply Universal Time (*UT*). The latter definition is ambiguous since *UT* is also used for *UT1*, another subcategory of universal time which even more closely follows the real motion of the Earth. Fig. E-1 shows historical differences between *TAI*, *UTC*, and *UT1*.

Terrestrial Time (TT)

Terrestrial Time, TT, is the current substitute for what was previously identified as terrestrial dynamic time, TDT, which in turn replaced the earlier ephemeris time, *ET*. Terrestrial time also uses the SI second as the unit of measure but is more precisely defined in terms of the dynamic equations of motion for the Earth. For most practical purposes, *TT* can be defined in terms of *TAI* by a simple offset, i.e., $TT - TAI = 32.184$ sec. (This offset is due to the historical origins of the time system. *TT* matched *ET* in 1984, when the use of *ET* was discontinued.) Consequently, for example, in January 1996, $TT - UTC = 32.184 + 30$ sec $= 62.184$ sec. Because atomic clocks have a small drift rate, changes between *TT* and *TAI* on the order of microseconds can accumulate over a period of years. (A more precise definition of terrestrial time based on the theory of relativity is given on the website.)

GPS Time

The GPS satellite system uses its own unique time called *GPS time*. Although presumably it could be accommodated with modern computer systems, the addition of a leap second is certainly inconvenient for GPS processing algorithms. Consequently, the GPS clock uses the SI second but does not introduce leap seconds. Like *TT* and *TAI*, *GPS time* maintains a fixed offset from *UTC* that changes by 1 sec whenever a leap second is

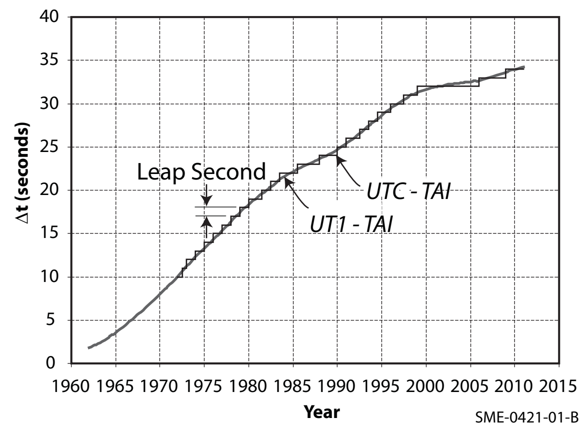

Fig. E-1. Historical Differences Between Universal Time and Atomic Time.

introduced in *UTC*. The difference between *GPS time* and *TAI* is constant: *GPS time* – *TAI* = 19 sec., except for a quantity of the order of tens of nanoseconds, varing with time. In order to allow *UTC* to be recovered from the time signals broadcast by GPS, the integral number of seconds by which the two differ is included in subframe 4 of the GPS navigation message. (See Fig. E-9 in Sec. E.2.) Consequently, a GPS receiver may provide time output which is either *GPS time* or *UTC*. The GPS system is becoming a common mechanism for time transfer between 2 locations on the Earth or between 2 satellites in low-Earth orbit. An error budget for *GPS time* transfer when both locations have a common GPS satellite in view is given in Table E-4.

Table E-4. Error Budgets for GPS Time Transfer (with Two Satellites in View). [Seidelmann, 2006]

Type of Delay	Best Case RMS (ns)	Worst Case RMS (ns)
Satellite Ephemeris	3	10
Ionospheric	2	100
Tropospheric	1	20
User-Position	1	1
Multipath	1	2
Receiver	1	1
Signal-to-Noise	7	1
***Total RMS** (for single 13-minute track)*	4.2	103
NOTE: Some of the errors depend on the distance (for example, the satellite ephemeris). The best case applies to distances of 2,000–3,000 km; the worst case is for distances of 6,000–8,000 km.		

E.3 Discontinuous Time

One of the most interesting characteristics of modern spacecraft is that updating the spacecraft clock, either from the ground or from GPS, means that time will appear to be discontinuous on board (see boxed example). Because the updates are typically very small, this makes little difference to time-tagging of on-board observations and similar activities.

Spacecraft and "Discontinuous Time"

For working problems in mechanics, time and the flow of time is one of our most basic and ingrained concepts. One of our most fundamental perceptions of time is that it is continuous and in some sense "smoothly flowing." This philosophical view of time may or may not be true in the extremes of general relativity or quantum electrodynamics, but it is certainly untrue for nearly all spacecraft clocks, due simply to the problem of synchronization.

Time is kept on board a spacecraft by a clock that drifts relative to the outside world. If we let the clock drift indefinitely it becomes increasingly difficult to make or interpret observations of the world. Accurate timing of observations is critical both for many scientific observations and also to correctly locate observations on the surface of the Earth or another planet. This implies a need to update spacecraft clocks from time to time by synchronizing them with the outside world using GPS, radio time signals, or a ground station clock synchronized with the worldwide time network.

Unfortunately, this implies a discontinuity or jump in time tagging of things like the spacecraft position as a function of time. For example, we may find ourselves in a position where the spacecraft is over the equator at exactly 12:00:00 and 100 km north of the equator at 12:00:01, such that the spacecraft appears to have made a jump in the space-time continuum. Even worse, we may find that the spacecraft is in two different places at the same time. Suppose that we cross the equator at 12:00:00. We reset the spacecraft clock backwards to 11:59:59 and then proceed along in the orbit such that we are now a few km north of the equator, once again, exactly at noon. While this is a perfectly reasonable sequence of events, remarkably few spacecraft data filters are prepared to handle discontinuous time.

Figure Eweb-1 shows the large-scale drift of the clock on the BREM-SAT spacecraft over a year. Figure Eweb-2 shows the fine drift on a day-to-day basis. With this particular clock one would need to introduce a correction averaging ~3 sec per day and ranging from +12 sec to –5 sec depending on the date. This corresponds to the spacecraft being ahead or behind by up to 90 km, even if the orbit propagation is perfect. Errors of this magnitude are certainly non-trivial in the interpretation of spacecraft data, including orbit and attitude information.

Both the lack of synchronization and the need to resynchronize clocks from time to time represent significant practical problems for spacecraft operations, and should be taken into account in both system and software design.

For simple time-tagging of data, this is not typically a problem. The synchronization error is simply one more component of the time-tagging error. The problem occurs whenever we need precise time differences or make use of the dynamic equations of motion for orbit, attitude, or payload data processing. In these processes, we must verify that the software will accommodate both "time jumps" and "time reversal."

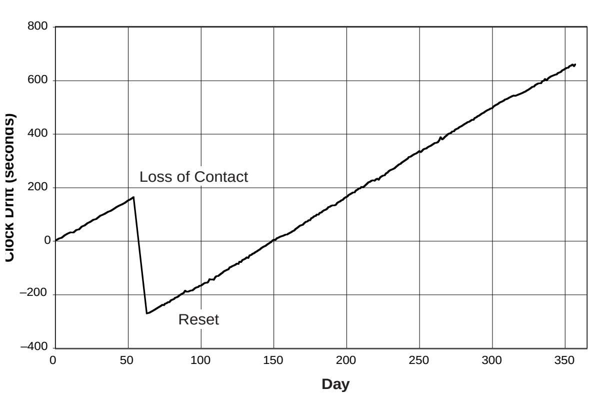

Fig. Eweb-1. Difference Between GPS Clock and BREM-SAT Computer Clock. This figure shows the cumulative large-scale drift of the BREM-SAT computer clock vs. a GPS clock over a year. The sharp drop shortly after day 50 reflects the loss of contact with BREM-SAT and subsequent reset after reacquisition.

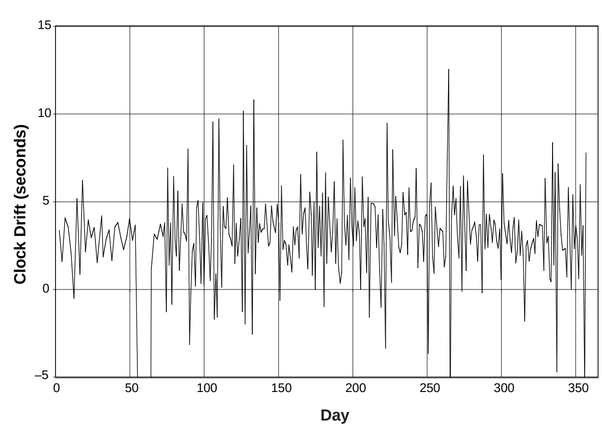

Fig. Eweb-2. Day-to-Day Difference Between GPS Clock and BREM-SAT Computer Clock. The figure shows fine drift of the BREM-SAT computer clock vs. a GPS clock on the ground. The reuslts are for each day, not cumulative.

However, spacecraft are also becoming more sophisticated and dynamic equations of motion are often used in data filtering and system modeling, such as the propagation of orbit and attitude states. Unfortunately, very few analytic systems take into account the cause or effect of time discontinuities. This has the potential of causing substantial problems for on-board software and was a key (and painful) issue in one of the spacecraft I (James Wertz) was involved with. The issue of "discontinuous time" is addressed in the adjacent boxed example. Anyone involved with time systems or high accuracy analysis of spacecraft data should be aware of this potential problem. So long as you are aware that "time" on the spacecraft can move forward, backward, or stand still over very brief intervals, finding solutions is typically not difficult—when they are planned in advance.

E.4 Solar Time, Sidereal Time, and Longitude on the Earth

Once we have launched a spacecraft into orbit and turned off the rockets, that orbit remains approximately fixed in inertial space, while the Earth continues to rotate beneath it. As described in Chap. 9, this means that the Earth underneath the spacecraft will change continuously, even though the spacecraft will repeat its path very nearly in inertial coordinates. To keep track of where the spacecraft is with respect to the surface of the Earth, we need to keep track of the rotational orientation of the Earth itself. This is done by knowing the *sidereal time*, which measures the rotation of the Earth relative to inertial space or the fixed stars. In contrast, civil time, such as that kept by a clock or radio signals, is a close approximation to *solar time*, which measures the rotation of the Earth with respect to the Sun. As shown in Fig. Eweb-3, solar time differs from sidereal time because of the motion of the Earth in its orbit around the Sun. When the Earth has made one rotation on its axis relative to the stars, it has also moved approximately 1/365th of the way around its orbit. Consequently, it takes about 4 min longer for the Earth to return to the same orientation relative to the Sun. Therefore, a solar day, or civil day of 24 hours, will be about 4 min longer than a sidereal day of about 23 hr 56 min. In order to quantify the relationship between sidereal time and civil time, we need a better understanding of solar time and the measurement of azimuth angles, both on the Earth and in the sky.*

The *celestial meridian* is the great circle in the sky passing through the celestial poles and the observer's *zenith*, i.e., the point straight overhead. As shown in Fig. Eweb-4, the *hour angle, HA*, is the azimuthal orientation of an object measured westward from the celestial meridian. As the Earth rotates eastward, an object on the celestial sphere (i.e., a star, planet, or the Sun) appears to move westward and its hour angle increases with time. It

* For definitions of rotation angles, the celestial sphere, and the globe plots in this Appendix see Chap. 8.

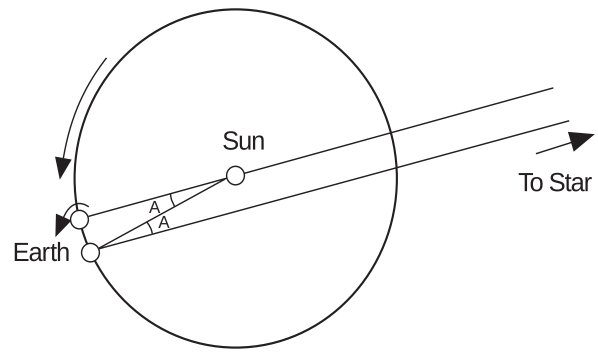

Fig. Eweb-3. Sidereal vs. Solar Day. The solar day (relative to the Sun) is about 4 min longer than the sidereal day (relative to the stars) because the Earth has moved in its orbit during the day and must rotate further to bring the Sun overhead again. The view shown is from the north ecliptic pole.

takes about 24 hours for an object to move completely around the celestial sphere, or about 1 hour to move 15 deg in *HA*; thus, 1 deg of *HA* corresponds to about 4 min of time. Therefore, the rotation of the Earth allows us to measure azimuth angles either in degrees or hours, minutes, and seconds, or equivalently to measure time as either hours, minutes, and seconds, or degrees. Because of the small difference between the solar and sidereal days, 1 deg actually differs slightly from 4 min. The correct transformation is:

$$\begin{aligned} 1 \text{ sidereal day} &= 86{,}164.1006 \text{ sec} \\ &= 360 \text{ deg} \end{aligned} \tag{Eweb-10a}$$

Therefore,

$$\begin{aligned} 1 \text{ deg} &= 239.344\,724 \text{ sec} \\ &= 3 \text{ min } 59.344\,724 \text{ sec} \end{aligned} \tag{Eweb-10b}$$

$$\begin{aligned} 1 \text{ sec} &= 0.004\,178\,074\,13 \text{ deg} \\ &= 0.000\,072\,921\,150 \text{ rad} \end{aligned} \tag{Eweb-10c}$$

$$\begin{aligned} 1 \text{ min} &= 0.250\,684\,448 \text{ deg} \\ &= 0.004\,375\,269\,00 \text{ rad} \end{aligned} \tag{Eweb-10d}$$

The *apparent solar time* is the local *HA* of the Sun expressed in hours, plus 12 hours. Thus, the Sun crosses the observer's celestial meridian at an apparent solar time of 12:00 noon. Apparent solar time is measured by a sundial. We could, for example, construct a simple sundial by driving a long nail perpendicularly through a flat piece of wood. For an observer in the northern hemisphere, if the nail is then pointed to the north celestial pole, the plane of the wood is parallel to the equatorial plane, and the angle relative to south of the nail's shadow on the wood is a measure of the *HA*, or the apparent solar time.

Due to the Earth's orbital motion, the Sun appears to move eastward along the ecliptic throughout the year. Because the Earth travels in an elliptical orbit it moves faster when near the Sun and slower when it is more distant. Therefore, the length of the solar day varies throughout the year.

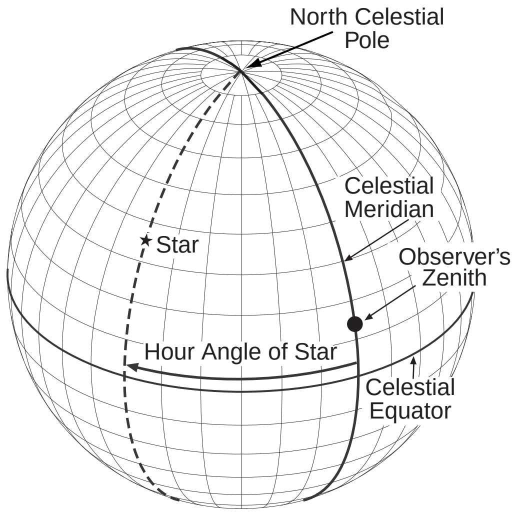

Fig. Eweb-4. Definition of Hour Angle. As the Earth rotates eastward, an object fixed in the sky appears to rotate westward and the hour angle increases.

However, even if the Earth were in a circular orbit with constant speed, the azimuthal component of the Earth's motion (parallel to the celestial equator) would vary, due to the inclination of the ecliptic relative to the equator. To illustrate this, consider a satellite in a nearly polar orbit as shown in Fig. Eweb-5. The satellite changes azimuth slowly while near the equator, and very rapidly while near the poles. Because the inclination of the ecliptic to the Earth's equator is only 23.5 deg rather than the large inclination illustrated in the figure, the variation in the length of the day due to the inclination of the ecliptic is small. The cumulative variation due to both eccentricity and inclination reaches a maximum of 16 min in November.

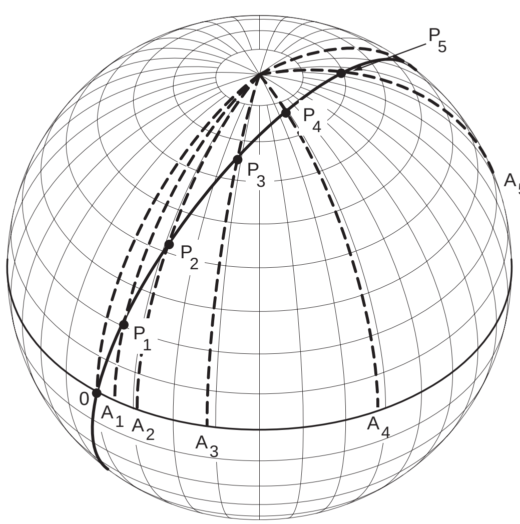

Fig. Eweb-5. Variation in Azimuthal Rate for a Satellite Moving Uniformly in its Orbit. $A_1, A_2, \ldots, A_5$ are azimuthal projections of the orbital points $P_1, P_2, \ldots, P_5$ and are equally spaced in time.

To provide a more uniform time than the real Sun, a fictitious *mean Sun*, which moves along the equator at a constant rate equal to the average annual rate of the Sun, has been introduced. *Mean solar time* is defined as the *HA* of the mean Sun. The difference between mean and apparent solar time is called the *equation of time*. This is often represented by an analemma, or figure 8, which frequently shows up in the middle of the Pacific ocean on globes of the world. This is the correction that is applied to a sundial to determine mean solar time.

In contrast to solar time, s*idereal time, ST*, is based on the rotation of the Earth relative to the stars and is defined as the *HA* of the vernal equinox, ♈.* The *local sidereal time*, *LST*, is defined as the local *HA* of ♈, *LHA* ♈. The *Greenwich sidereal time*, *GST*, (also called the *Greenwich HA of the vernal equinox*, *GHA* ♈) is the hour angle of the vernal equinox for an observer on the Earth's prime meridian, which goes through the Royal Greenwich Observatory.

The *right ascension*, *RA*†, of any star or other celestial object is the azimuthal component of the star's position measured eastward from ♈. It is the celestial equivalent of longitude measured on the surface of the Earth.

Figure Eweb-6 shows the azimuthal angular relationships between ♈, a star or other celestial object, the Greenwich meridian, and the local meridian of the observer. All of these are azimuthal angles projected onto the celestial equator. From this figure we can determine a variety of relationships. For example, the *local sidereal time*, *LST*, may be determined from the *RA* and observed *HA* of any star:

$$LST = LHA\ \Upsilon = [LHA^* - RA^*]\text{mod}_{24\text{ hrs}} \qquad \text{(Eweb-11)}$$

where LHA^* and RA^* are the *HA* and *RA* of the star, both converted to time and $\text{mod}_{24\text{ hrs}}$ is the modulo function, which expresses the result as a quantity between 0 and 24 hr. In the example in Fig. Eweb-6, LHA^* is 135 deg

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* The *vernal equinox* is the location of the Sun in the sky on the first day of spring. It is located at one of the two intersections between the ecliptic plane and the celestial equator. The symbol for the vernal equinox (♈) is the astronological symbol for the constellation Aries, the Ram, because the vernal equinox is also called the First Point of Aries. (The vernal equinox was in Aries when the name was given to that region of the sky.)

† The units here can be particularly confusing. Throughout this book we use deg and decimal fractions of a deg for most angular measurements, although radians will occasionally be used. In astronomical work, angles, and particularly declination (the latitude-like measurement on the celestial sphere), are often measured in deg, min, and sec of arc, which are written as, 10°, 10′, and 10″, where 1 min of arc is 1/60th deg, and 1 sec of arc is 1/60th of a min. Very small angles such as the resolution of optical instruments, are frequently measured in sec of arc. Because right ascension also corresponds to time, it is often measured in hours, min, and sec, written as 10^h, 10^m, 10^s. Here 1 hr corresponds to 15 deg, 1 min corresponds to 0.25 deg, and 1 sec is 1/60th of a min, or 1/240th of a deg. Thus, min and sec of right ascension are **not** the same as min and sec of declination. We avoid this confusion by using deg and decimal fractions of a degree for all angles.

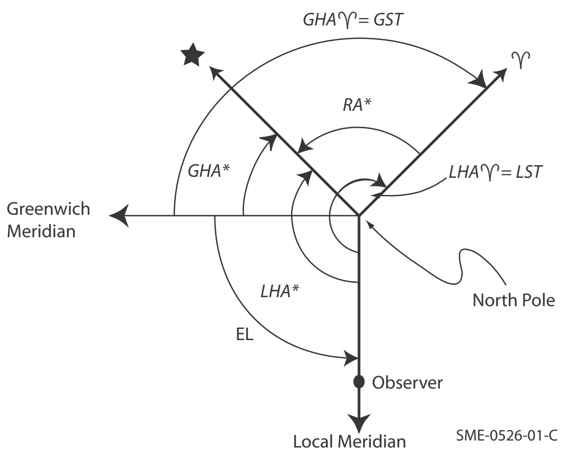

Fig. Eweb-6. Relationship Between Local Position on the Earth, Hour Angle, and Sidereal Time. The view is looking down from the Earth's North Pole.

or 9 hours, *RA** is –90 deg or –6 hours and *LST* is 15 hours. Similarly, we can determine the *Greenwich sidereal time, GST,* from:

$$GST = GHA\,♈ = [GHA^* - RA^*]\mathrm{mod}_{24\ \mathrm{hrs}} \qquad \text{(Eweb-12)}$$

*GHA** is the *GHA* of any star, converted to time. Again in the example figure, *GHA** is 45 deg or 3 hours; thus *GST* is 9 hours. Note that the sidereal time at Greenwich is equal in magnitude to the right ascension of the Greenwich meridian. The difference between *GST* and *LST* (6 hours in the example of the figure) corresponds to the observer's east longitude (90 deg in the example). In general,

$$LST = GST - EL/15 \qquad \text{(Eweb-13)}$$

where *EL* is the observer's east longitude in deg and *LST* and *GST* are in hours.

From the definition of mean solar time, the Greenwich mean time, *GMT*, or universal time, *UT*, equals the *GHA* of the fictitious mean Sun plus 12 hours:

$$UT = GMT = GST - RA_0 + 12\ \text{hours} \qquad \text{(Eweb-14)}$$

where RA_0 is the right ascension of the mean Sun. For a given *UT* on any calendar date, the expression for the *GST* is given by

$$\begin{aligned} GST &= RA_0 - 12\ \text{hours} + UT \\ &= 6^h 38^m 45^s.836 + 8{,}640{,}184^s.542T \\ &+ 0^s.092\,9T^2 + UT \end{aligned} \qquad \text{(Eweb-15)}$$

where *T* is the number of Julian centuries of 36,525 days which have elapsed since noon (GMT) on January 0, 1900. The corresponding equation for *GST* expressed in degrees is

$$\begin{aligned} GST &= 99°.690\,983 + 36{,}000°.768\,925T \\ &+ 0°.000\,387T^2 + UT \end{aligned} \qquad \text{(Eweb-16)}$$

where *UT* is in degrees and *T* is again in Julian centuries.

The Julian date, *JD*, defined in Sec. E.1 provides a convenient mechanism for determining *T* in Eqs. (Eweb-15) and (Eweb-16). The *JD* for Greenwich mean noon on January 0, 1900 (i.e., for January 0.5 1900) is 2,415,020.0. The *JD* for any date in the current era may be obtained by adding the day number of the year to the date *JD* for January 0.0 *UT* of that year as listed in Table E-1. For example, to find the *GST* for 3 hours *UT* on July 4, 2026:

Day number of July 4.125 (3^h *UT* July 4)	186.125
+ *JD* for January 0.0, 2026	+2 442 777.500
= *JD* for July 4.125, 2026	= 2 442 963.625
– *JD* for January 0.5, 1900	– 2 415 020.000
= *T* in days	27 943.625
÷ 36,525 =*T* in Julian centuries	0.765054757
8,640,184.542 T+0.0929 T^2	6 610 214.340 sec
	$=76^d 12^h 10^m 14^s.340$
+ first term Eq. (Eweb-15)	6 38 45.836
+ *UT*	45.00
GST	$=21^h 49^m 0^s.176$

Given the Greenwich sidereal time, we can finally compute the longitude of the subsatellite point for a spacecraft whose ephemeris is known. Specifically from Eqs. (Eweb-11 and Eweb-13) we have

$$EL_{spc} = RA_{spc} + GST\ \text{(in degrees)} \qquad \text{(Eweb-17)}$$

where RA_{spc} is the right ascension in degrees of the spacecraft at the time in question and *EL* is the East longitude of the subsatellite point.

Note that the Greenwich sidereal time defined by these equations is a uniformly flowing time whereas the actual rotation of the Earth on its axis has a very low amplitude wobble as described in the front of the appendix. Consequently, the accuracy of the resulting longitude from the general equations above will be about 0.005 deg (= 500 m at the equator) if the spacecraft ephemeris is known precisely.

E.5 Relativistic Time

On the whole, relativity is not something to worry about in space missions, except for spacecraft designed specifically to test relativistic theories. Nonetheless, we need to understand the boundaries of Newtonian physics so that we can understand the magnitude of the errors that occur and under what circumstances they become relevant. Consequently, this section summarizes the relativistic effects most important for work with spacecraft clocks and the level of error that results when relativistic effects are ignored. A mathematical summary of relativistic time as it applies to the generation of ephemerides is given by Seidelmann [2006]. A detailed mathematical and physical explanation of most effects in special and general relativity is provided by Misner et al. [1973]. A detailed explanation of relativistic effects applied to spacecraft and an extensive reference list are given by Ashby and Spilker in Parkinson and Spilker [1996]. However, this volume does not cover the relativistic time systems adopted by the IAU which are discussed by

Seidelmann [2006]. More popular and readable explanations of the physical phenomenon in special and general relativity are provided by a wide variety of texts including, for example, Einstein [1950], Mermin [1968], and Kaufmann, III [1973].

The fundamental relativistic effect that is important for time systems is that the rate at which time flows depends on the observer. In *special relativity* the most important effect, known as *time dilation*, is that moving clocks run slow, i.e., any clock in motion with respect to me runs slower than my clock. Here the concept of a "clock" is a very general way of expressing the flow of time, and applies to clocks as fundamental as those which are inside an atom. One of the most compelling demonstrations of time dilation is associated with the radioactive decay of atomic particles. When cosmic rays interact with the Earth's upper atmosphere, they produce streams of very high velocity mesons. These mesons undergo radioactive decay, but do so far more slowly than do mesons produced in experiments "at rest" with respect to us on the surface of the Earth.

In general, if we have a collection of observers in motion with respect to each other (for example, observers with very precise clocks on a collection of spacecraft in different orbits) each observer will determine that all of the other observers' clocks run slow. (It is not just that they **see** the clocks run slow due to the finite velocity of light. They take the velocity of light into account and **calculate** that all of the other clocks actually are running slow.) This leads to a wide variety of apparent paradoxes, in terms of different observers comparing the results of timing experiments. Most of these are resolved by a second important phenomenon in relativity—the *lack of absolute simultaneity*, which states that whether two distant events are simultaneous (occur at the same time) does not have an absolute meaning, but depends upon the state of motion of the observer.

In the *general theory of relativity*, an effect similar to time dilation is that clocks run slower in a gravitational field.* Thus, clocks on the surface of the Earth run more slowly than those away from any of the planets, even if they are not moving relative to the Earth. Similarly, clocks run slower as they approach any massive object.

Time dilation, the slowing of clocks in a gravitational field, and related relativistic effects are discussed in detail in the references above. The important issue for spacecraft is that the flow of time in its most basic sense is not an absolute thing but depends on both the gravitational field and the state of motion of the clock. Clocks in orbit, even atomic clocks, don't behave like clocks on the surface of the Earth, although the differences are remarkably small.

* Effects in general relativity are tied to those in special relativity by the *Principle of Equivalence* which states that the effects of a gravitational field are equivalent to those of an accelerating frame of reference. This also implies that a freely falling, non-rotating body is equivalent to an accelerating one which is not in a gravitational field at all. Thus, if an object has fallen from infinity to near the surface of a gravitating body, its "clock" is equivalent to an accelerating one that is not in a gravitational field. Therefore, the slowing of clocks in a gravitational field is equivalent to the time dilation of a "stationary" clock near the celestial body when compared with one which is moving at the velocity it would have when falling from being at rest at infinity (i.e., the escape velocity of the celestial body).

Table Eweb-2. Typical Relativistic Time Effects. Relativistic effects are not important for normal spacecraft operations, but may become relevant if extreme precision is required. Note that for the Doppler shift (2nd row), the first term is the ordinary (Newtonian) Doppler shift and the second term is the relativistic correction.

Effect	Basic Formulas	Examples
Time Dilation (Moving clocks run slow)	Time delay: $\frac{\Delta t}{t} = \frac{1}{\sqrt{1-v^2/c^2}} \approx \frac{1}{2}\frac{v^2}{c^2}$	In Low-Earth orbit = 11 ms/yr In Earth's orbit about Sun = 158 ms/yr On Fastest Spc (60 km/s) = 632 ms/yr
	Doppler shift (frequency shift) due to spacecraft velocity: $\frac{\Delta f}{f} \approx \frac{v}{c}\cos\theta + \frac{1}{2}\frac{v^2}{c^2}$	Effect on signal at 10.23 MHz (GPS): In Low-Earth orbit = 269.747 + 0.004 Hz In Earth's orbit about Sun = 1023.708 + 0.051 Hz On Fastest Spc = 2047.416 + 0.205 Hz
	Sagnac effect (time delay due to rotating frame): $\frac{\Delta t}{t} \approx \frac{\omega^2 R^2}{2c^2}$	On Earth's surface = 0.038 ms/yr On Jupiter's surface = 28 ms/yr
Gravitational Time Dilation (Clocks run slow in a gravitational field)	Gravitational frequency shift: $\frac{\Delta f}{f} \approx \frac{\mu}{Rc^2}$	Effect on signal at 10.23 MHz (GPS): At surface of Earth = 0.007 Hz At surface of Jupiter = 0.202 Hz At surface of Sun = 21.702 Hz At Earth distance (due to Sun's gravitational field) = 0.101 Hz

Definition of Variables:

c = velocity of light (299,792,458 m/s)
v = velocity of clock relative to the observer
θ = angle between velocity vector and line of sight
R = radius of the celestial body
μ = GM = gravitational constant of the celestial body
ω = angular rotation rate
t = time
Δt = time difference
f = frequency
Δf = frequency shift

As summarized in Table Eweb-2, relativistic effects are dramatically small and are of no consequence for most space missions. For example, a clock on a spacecraft in low-Earth orbit, moving at 8 km/s relative to an observer on the Earth, runs slow by 11 msec per year or a fractional shift of 3.5×10^{-10}. A clock sitting on the "surface" of Jupiter runs slow by 1 part in 5×10^7 which changes a 10 MHz signal from a spacecraft there by 0.2 Hz.

The effects of relativity are extremely small, even by the standards of precise spacecraft measurements. Nonetheless, they are measurable and do have an impact on our basic understanding of time systems and, perhaps more importantly, what we mean by time. In 1991 the general theory of relativity was explicitly adopted as the theoretical framework for defining space-time reference frames.* The implication is that time is no longer defined in an absolute sense, i.e., as so many ticks of an atomic clock, but only as so many ticks **in a specific coordinate frame**. The key issue for spaceflight is that no matter how you measure it—the decay of subatomic particles, the clicks of a Cesium clock, or the number of old movies you can watch—time flows at a different rate on board spacecraft than it does on the Earth. We need to tie our definition of time to the coordinate frame which we are using. This in turn leads to the 2 remaining types of time, both of which fall into the category of dynamic time, or time which depends on the orbital motion in the Solar System.

Terrestrial Time (TT)

The fundamental unit of *Terrestrial Time* is the *SI* second, as kept by a perfect atomic clock at mean sea level on the surface of the Earth. Here the second is the same as that defined by International Atomic Time, *TAI*, but is given a more precise definition by being attached to a specific reference frame. Because the units are identical, *TT* is equal to *TAI* time plus an offset of 32.184 sec. The offset comes about for historical reasons having to do with the evolution of time systems. In a series of actions from 1950 to 1958, *Ephemeris Time*, *ET*, was adopted by the international community as the most fundamental definition of time based on the motion of the planets as defined by Simon Newcomb's tables of the Sun published in 1900. In 1967, the atomic clock definition of the second was introduced, which led to very small variations in the rate of time and small offsets in differently defined time systems. In 1984, *ET* was abandoned in favor of *terrestrial dynamic time*, *TDT*. Finally, in 1991 when relativity was adopted as the appropriate analytical framework, *TDT* was renamed *terrestrial time*, *TT*. This is currently the time scale in use for generating ephemerides for the motion of celestial objects as seen from the surface of the Earth.

Barycentric Dynamic Time (TDB)

Barycentric dynamic time is the independent variable in the orbital equations of motion with respect to the barycenter of the solar system. Thus, *TDB* transforms time as measured on the surface of the Earth to time as kept by the motion of the planets (thus, in a different coordinate frame). *TDB* and *TT* are very close to each other (i.e., to within less than 2 msec), with the differences between them being determined by means of mathematical expressions. An approximate formula for converting *TT* to *TDB* that is sufficient for all practical applications is:

$$TDB = TT + 0.001\,658 \sin g + 0.000\,014 \sin 2g \qquad \text{(Eweb-18)}$$

where the times and coefficients are in SI seconds and g, expressed in deg, is given by

$$g = 357.53 \text{ deg} + 0.985\,600\,3\,(JD - 2{,}451{,}545.0) \qquad \text{(Eweb-19)}$$

and *JD* is the Julian date expressed to two decimals of a day.

References

Einstein, Albert. 1950. *Relativity: The Special and the General Theory*. Translated by Robert W. Lawson. NY: Crown Publishers.

Fliegel, Henry F. and Thomas C. Van Flandern. 1968. "A Machine Algorithm for Processing Calendar Dates." *Communications of the ACM*, Vol. II, p. 657.

Kaufmann III, William J. 1973. *Relativity and Cosmology*. NY: Harper and Row Publishers.

Mermin, N. David. 1968. *Space and Time in Special Relativity*. NY: McGraw-Hill.

Misner, Charles W., Kip S. Thorne, and John Archibald Wheeler. 1973. *Gravitation*. San Francisco: W.H. Freeman and Company.

Parkinson, B. W., J. J. Spilker Jr., P. Axelrad, and P. Enge (eds.) 1996. *Global Positioning System: Theory and Applications*, Vol. 1, p. 183. Washington, D.C.: American Institute of Aeronautics and Astronautics.

Seidelmann, P. Kenneth (ed.) USNO. 2006. *Explanatory Supplement to the Astronomical Almanac*. Mill Valley, CA: University Science Books.

* For both historical and practical reasons, the astronomical community has been the caretaker of fundamental clocks and the definition of time. The definitions of time systems given here and Sec. 4.1.1 are those adopted by resolutions of the general assembly of the International Astronomical Union.